Basic Concepts

The four basic circles and how to locate things

Terrestrial Equator: The great circle perpendicular to the pole of rotation of the earth.

Terrestrial Longitude: The angular distance of a point on the earth projected onto the terrestrial equator. It is measured from a favored location (e.g. Alexandria). Since the end of the last century this is measured from Greenwich. In the nineteenth century, Paris was a rival. This is irrelevant to our story. Longitude could not be measured accurately in the ancient world. Ptolemy's Geography attempts to provide longitude using estimates of distance and eclipses.

Terrestrial Latitude: The angular distance of a point on the earth from the terrestrial equator. Latitude can be measured accurately by a simple experiment. If one knows the distance of the highest altitude h (see below) of the celestial equator (see below), then the latitude is 90° - h. There are many ways of doing this. For example, take a star whose declination δ (see below) is known. Measure its highest altitude h in the middle of the night (or take the sun at dead noon, preferably on the day of an equinox (see below), which $\delta = 0°$), and the terrestrial latitude is $90° - (h - \delta)$.

The Horizon: the circle where the sky and the earth (conceived as lacking undulation) appear to meet.

Altitude: The altitude is the angular distance of a point to the horizon (in degrees). 90° altitude is the **zenith**.

Azimuth: The angular distance of a point projected onto the horizon. Usually (but not always) measured from the northern point on the horizon clockwise (in degrees).

Celestial Equator: The projection of the Terrestrial Equator onto the sky.

Declination: The angular distance of a point from the Celestial Equator (normally measured in degrees).

Right Ascension: The angular distance of the projection of a point onto the Celestial Equator from the Vernal Equinox (normally measured in hours from west to east).

Declination and right ascension play only a secondary function in ancient astronomy. Copernicus made it primary, and so it has been since.

Ecliptic Circle: The apparent path of the sun in the course of one solar year.

Obliquity of the Ecliptic: the angle of the ecliptic to the celestial equator. In fact, it changes slightly over time (nutation).

Equinox (night equals day)**:** The point of intersection of the ecliptic circle and the celestial equator. The sun's crossing the vernal equinox marks the beginning of Spring. The sun's crossing the autumnal equinox marks the beginning of autumn. On the equinox the day equals the night.

Tropic (turning) or **Solstice** (stopping of the sun)**:** The most northern and southern points on the ecliptic. Summer begins when the sun is passes the nothern tropic or summer solstice (i.e. stops going north and turns to go south), and winter when it passes the southern tropic or winter solstice (i.e. stops going south and turns to go north). The longest day north of the equator is on the days of the summer solstice, and the shortest on the day of the winter solstice.

Latitude: The angular distance of a point projected onto the ecliptic circle.

Longitude: The angular distance of the projection of a point onto the Celestial Equator from the Vernal Equinox (normally measured in degrees from west to east; however, in identifying a position, the Babylonians and Hellenistic Greeks would normally cite the sign (1/12 of a circle) and the number of degrees from the beginning of the sign).

Years:

Sidereal or Zodiacal Year: The time it takes for a planetary body to return to the same declination or latitude (it makes not difference) or to conjunction with the same star.

Tropical Year: The time it takes for the sun to return to the same topic. Note that the tropical year of the sun and its zodiacal year are different becausseof precession, which Hipparchus discovered about 150 BCE.

Synodic Year: The time it takes for a body to return to the same phase.

The Basic Rule for Calculating Synodic Periods:

Suppose that two bodies b_1 and b_2 revolve about some common center (the earth or the sun or whatever) that that their respective periods of revolution are p_1 and p_2 where $p_1 > p_2$ (i.e., b_1 revolves more slowly than b_2). Hence, the amount of motion in a time t of b_1 and b_2 will respectively be: t/p_1 and t/p_2. Hence, where s_{12} is the synodic period of the two bodies, the difference in their motion will be the synodic motion of the two bodies, $t/s_1 = t/p_2 - t/p_1$. Hence, $t/p_1 + t/s_{12} = t/p_2$.

1. Hence $s_{12} = \dfrac{p_1 * p_2}{p_1 - p_2}$.

If b_2 is the sun, and p_2 is measured in years, then $p_2 = 1$, and

2. $s_1 = \dfrac{p_1}{p_1 - 1}$.

Let b_1 be a planet and b_2 be the sun, and let b_1 revolve Z times in Y years (i.e., Y number of revolutions of the sun), and let S be the number of synodic cycles in this period.

3. $S + Z = Y$

Next let b_1 be the sun and b_2 the moon, and let the moon revolve Z times in Y years, with S the number of synodic cycles (synodic months).

4. $Y + S = Z$

Finally, suppose that the universe is heliocentric and that b_1 is Mercury or Venus and that Z is the number of rotations of one of these about the sun in Y years (now rotations of the earth about the sun). Here, S is the number of synodic cycles between the earth and the other planet. Since the earth's period is less,

5. $S + Z = Y$

Given that in a geocentric universe, the sun's zodiacal period is the same as that of Venus or Mercury, this is the only way one can explain the synodic period. Hence, this explanation was unavailable before Copernicus.

Examples:

Mercury has a heliocentric zodiacal period of 88 days. Hence, its synodic period is:

$1/s = 1/88 - 1/365.25$, so that $S = 116$ days

Saturn has a zodiacal period of 29 1/2 years, or makes 2 cycles in 59 years. Hence, it makes 59 - 2 = 57 synodic cycles in 59 years. Hence s = 59/57 *365.25 days = 378 days.

Basic Ancient Concepts:

Planetary Star: Any of the sun, moon, or planets (the Greeks use the word πλανητης to refer to the planets alone or to planetary stars)

Planet: Venus and Mercury (**isotachic planets**) and Mars, Jupiter, and Saturn (**slow planets**). In Ptolemy and later astronomy, the isotachic planets are **inner**, and the slow planets are **outer**.

Celestial Equator: The equator of the sphere of the fixed stars which rotates once daily.

Ecliptic Circle: The path of the sun in later Greek astronomy, merely of the center of the zodiac in earlier Greek astronomy. Since in Babylonian planetary theory, it is usual not to discuss latitude, it is less clear how the ecliptic circle would have been understood. The planetary stars all move west/east near the ecliptic circle.

Phase: Generally, any of several cyclical phenomena associated with a synodic period, e.g., syzygy, last evening setting.

Syzygy: An alignment of a given star, the earth and the sun (or some other star). We need to distinguish the syzygies of fixed stars, slow planet sand the moon and the syzygies of an isotachic planet.

Fixed Stars, the Moon and Slow Planets:

Opposition: The earth is between the sun and the star.

Conjunction: The sun and the planetary star are 'together'. For Ptolemy and later astronomers, the alignment is earth-moon-sun or earth-sun-slow planet.

Isotachic Planets (equally fast): the planet is seen to oscillate on each side of the sun, when it is behind it appears in the morning, and when it is ahead it appears in the evening.

Evening to Morning Conjunction (epicyclic models and later: **Near Conjunction**): The planet is moving from its appearance in the evening to its appearance in the morning. On the epicycle model (and Copernican), the planet is nearest to the earth at this conjunction.

Morning to Evening Conjunction (epicyclic models and later: **Far Conjunction**): The planet is moving from its appearance in the morning to its appearance in the evening. On the epicycle model (and Copernican), the planet is furthest from the earth at this conjunction.

Settings and Risings: Stars between the local Arctic and Antarctic circles rise every day in the east and set in the west.

Invisibility Periods and Settings and Risings as Phases (fixed stars): Since the sun moves in relation to the fixed stars (which don't move at all--ignoring a phenomenon known as precession), it passes them. For the fixed stars this is once every solar year, since the synodic year equals the solar year. When the star is near the sun, it cannot be seen because the sun is too bright. Let us picture the situation more clearly.

1. The star is visible in the evening a little after sunset.
2. The sun is moving east, so that each night the star is closer to the sun. This means that the star appears closer to the horizon.
3. The star now appears for the last time in the evening (**Last Evening Setting**).
4. Now the star is in the light of the sun and cannot be seen.
5. **Conjunction** (note that conjunction is not observable).
6. The sun is passing the star.
7. The star now appears in the early morning just before sunrise (**First Morning Rising**).

The period between Last Evening Setting and First Morning Rising is the **Invisibility Period.**

8. As the sun overtakes the star further, the star gradually appears higher in the sky (westward) before sunrise.
9. The star also is seen as rising just after sunset in the east for the first time (**First Evening Rising** or **Acronychal Rising**).
10. The star is now at **opposition** to the sun.
11. A brief time later the star will appear in the west for the last time in the morning (**Last Morning Setting** or **Acronychal Setting**).
12. Now the star does not appear in the morning but is seen in the evening.

Warning: This is the order for stars near the ecliptic. For stars away from the ecliptic the order is different.

For northern stars that have some invisibility period, the order is:
Last Evening Setting / True conjunction / First Evening Rising / Last Morning Setting / True Opposition / First Morning Rising

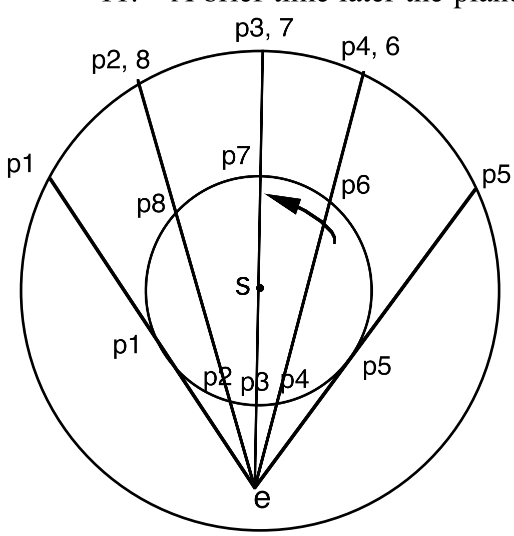
For southern stars that have some visibility period, the order is:
Last Evening Setting / True conjunction / Morning Setting / Evening Rising

Invisibility Periods and Stations, Settings and Risings as Phases, Retrograde Motion (slow planets): Since the sun moves faster than the slow planets, it passes them. For the slow planets this is once every synodic year. When the planet is near the sun, it cannot be seen because the sun is too bright. Also, when the planet is near opposition to the sun, it stops moving and starts to move in the other direction until after opposition (**retrograde motion**), when it stops and again proceeds forward. Let us picture the situation more clearly.

1. The planet is visible in the evening a little after sunset.
2. The sun is moving east, so that each night the planet is closer to the sun. This means that the planet appears closer to the horizon.
3. The planet now appears for the last time in the evening (**Last Evening Setting**).
4. Now the planet is in the light of the sun and cannot be seen.
5. Conjunction.
6. The sun is passing the planet.
7. The planet now appears in the early morning just before sunrise (**First Morning Rising**).

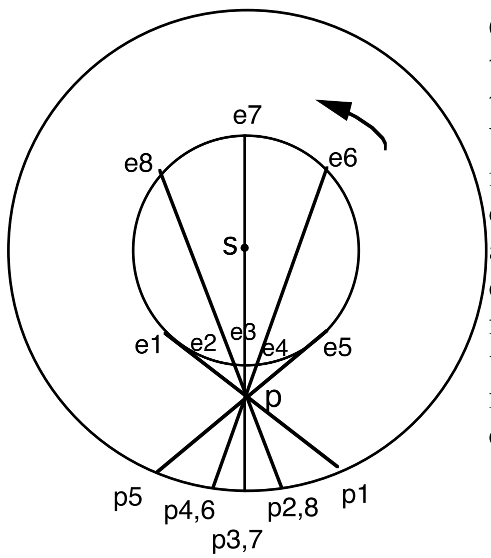
The period between Last Evening Setting and First Morning Rising is the **Invisibility Period.**

8. As the sun overtakes the planet further, the planet gradually appears higher in the sky (westward) before sunrise.
8. Just before the next phase, the planet stops (**First Station**) and begins to move backwards (**Retrograde Motion**).
9. The planet also is seen as rising just after sunset in the east for the first time (**First Evening Rising** or **Acronychal Rising**).
10. The planet is now at **opposition** to the sun.
11. A brief time later the planet will appear in the west for the last time in the morning (**Last Morning Setting** or **Acronychal Setting**).
11. Soon after Last Morning Setting, the planet stops again (**Second Station**) and begins to move forward againg.
12. Now the planet does not appear in the morning but is seen in the evening.



The inner planet p moves counter-clockwise, the earth is fixed at e.

Mercury, Venus, Mars, Jupiter, and Saturn all appear to move in the same direction with regard to the fixed stars as the sun. Then they stop (first station), move backwards for a shorter period (retrograde), stop again (second station), and proceed forwards. In the heliocentric model, retrograde motion arises from the fact that the planets do not move around the sun at the same angular speed. This occurs because the outer planets move slower than the earth and the inner move faster. The angles of observation for outer planets illustrates the different direction of movement.



While the earth moves, the planet fixed at p appears to oscillate.

To determine retrograde motion for inner planets from a Copernican model, since they move faster than the earth, think of the earth as fixed. A planet (p) would then appear to oscillate with regard to the sun (s) against the background of the fixed stars (the outer circle). Its normal movement is counter-clockwise, and its retrograde clockwise. One may observe, therefore, that there will be a retrograde every time an inner planet is in near conjunction with the sun. Since the earth and the planet in fact move counter-clockwise, from a geocentric point of view, the sun and the inner planet will appear to move forward west to east, with the planet occasionally moving east to west in retrograde.

For the outer planets, since they are slower, think of them as fixed and the earth as moving counter-clockwise. If one then draws a line from the earth (e) through an outer planet (p) to the stars, one will see that it will appear to oscillate against the background of the fixed stars (the outer circle). When the planet is in opposition to the sun (s), it will appear to move backwards (clockwise). Since the planet in fact also moves counter-clockwise according to the diagram, the actual apparent motion will somewhat*approximate* the angular motion of the planet around the sun plus the apparent motion of the planet if it were fixed.[1] Hence, the sun and the planet will normally move west to east, but occasionally the planet will move east to west in retrograde. Thus for every planet, there will be one retrograde for every return of the planet to the same position relative to the sun. This period will be a function of the earth's and the planet's motion. It is, therefore, convenient to define the synodic year of a planet as the amount of time it takes for a planet to return to conjunction or to opposition. This is called the synodic year:

Planet	Synodic Period	Zodiacal Period (Tropical Years)
Mercury	116 days	1 year
Venus	584 days	1 year
Mars	780 days	1 year 158 days
Jupiter	399 days	11 years 300 days
Saturn	378 days	29 years 158 days

It is obvious that a retrograde will occur once every synodic year (i.e. either once a far conjunction or once an opposition). The only anomally in the Eudoxan values is Mars, where Eudoxus will predict a retrograde (and opposition) almost exactly three times in the actual synodic year, and hence predicts three retrogrades in a period in which only one will occur. One reason for this will be apparent.

[1] How closely will depend on the relative sizes of the orbits and the ratio of the periods of orbits. This, of course, also assumes that the orbits are circular and regular about a common point.

Invisibility Periods and Stations, Settings and Risings as Phases, Retrograde Motion (isotachic planets): Since the sun moves together with the isotachic planets, they oscillate on each side of the sun. One complete oscillation is one synodic year. When the planet is near the sun, it cannot be seen because the sun is too bright. When the sun is overtaking the planet, not only does the planet move slower than the sun, but it actually moves backwards for a brief period (**retrograde motion**). Let us picture the situation more clearly.

1. The planet is at the furthest distance in the evening from the sun (**Maximum Evening Elongation**). The planet is visible in the evening for some time after sunset (under 3 hours for Venus; for Mercury less than an hour, but sometimes Mercury is not visible at all).
2. The planet is visible in the evening a little after sunset.
3. The planet now stops (**First Station** or **Evening Station**).
4. The planet is now moving backwards (**Retrograde Motion**).
5. The planet now appears for the last time in the evening (**Last Evening Setting**).
6. Now the planet is in the light of the sun and cannot be seen.
7. **Evening to Morning Conjunction** or **Near Conjunction**.
8. The sun is passing the planet.
9. The planet now appears in the early morning just before sunrise (**First Morning Rising**).

The period between Last Evening Setting and First Morning Rising is the **Evening to Morning Invisibility Period**.

10. The planet now stops (**Second Station** or **Morning Station**).
11. The planet proceeds to move forward again, but not as fast as the sun.
12. The planet is now at its furthest distance from the sun (**Maximum Morning Elongation**). The planet is visible in the morning for some time before sunset (under 3 hours for Venus; for Mercury less than an hour, but sometimes Mercury is not visible at all).
13. Now the planet starts to move faster than the sun. This means that it is now overtaking the sun. This means that the time before sunrise when the planet is seen is less and less.
14. The planet now appears for the last time in the early morning just before sunrise (**Last Morning Rising**).

The period between Last Evening Setting and First Morning Rising is the **Morning to Evening Invisibility Period**.

15. The planet is now in the light of the sun and cannot be seen.
16. **Morning to Evening Conjunction** or **Far Conjunction**.
17. The planet is now moving ahead of the sun.
18. The planet now far enough ahead of the sun that it appears for the first time in the evening (**First Evening Setting**).
19. The planet continues to move ahead of the sun so that it appears more and more in the evening.

Warning: Mercury has its retrogade motion during the invisibility period, i.e. 3 and 4 occur after 5 while 10 and 11occur before 9. Also since Mercury often does not get far enough from the sun to appear, 18 and 5 or 9 and 14 often not occur at all.

Warning: The zodiacal movements of the sun and the planets are irregular, so that these phases do not occur at the same intervals. This is due to the fact that the motions of the planets (including the earth) are elliptical in accordance with Kepler's first and second laws.

Lunar Phases: These are the most familiar, but four phases are particularly significant.

1. **New Moon**: The first appearance of the moon near sunset. The moon is to the east of the sun. The beginning of the month in Babylon, Greece, and later Islam
2. **Full Moon**: The moon is in opposition to the sun.
3. **Last Moon**: The moon appears for the last time in the morning as it catches up with the sun. The beginning of the month in the Egyptian religious calendar.
4. **Conjunction** (also called **New Moon**):The moon and sun are together.

Warning: The movements of the sun and the moon are irregular, so that these phases do not occur at the same intervals. This is due to the fact that the motions of the moon and the earth are elliptical in accordance with Kepler's first and second laws.

Lunar Cycles:

Synodic Month: This is the month that is somewhat familiar to us.

Zodiacal Month: This is the period in which it takes the moon to return to the same longitude (or conjunction with the same fixed star).

Let S be the number of synodic months in a given period, exactly Y years.

Let Z be the number of zodiacal months in the same period.

THEN S + Y = Z.

LET S = 235 synodic months and Y = 19 years. Then Z = 254.

Now Y = 19 * 365 1/4 days = 6939 3/4 days.

Hence, one zodiacal month = Y/Z days = 27.32185 days = 27;19,18,39,41,6,9 on average!!!

And, one synodic month = Y/S days = 29.530851 days = 29;31,51,3,49,47,14 on average!!!

Draconitic Month: The moon also moves about 5° from the ecliptic. If we are interested in predicting eclipses this is crucial. The width of the moon is about 1/2°, while the shadow of the earth on the moon is less than 2°. Hence, eclipses will only occur when the moon is very near the ecliptic.

Node: The two points where the orbit of the moon intersects the ecliptic. If the moon is comes from below the ecliptic, the node is the **ascending node**, and if the moon comes from above the ecliptic, the node is the **descending node**.

The period of the moons return to the same node is the **Draconitic Month**.

If S = 223 synodic months, D = 242 dracontic months.

Hence, one draconitic month = S * average synodic month/D =
27;12,44,19,38,41 days

Anomalistic Month: The moon has an irregular motion. This month is period of the return of the moon to the same speed in the same direction.

If S = 223 synodic months, A = 239 anomalistic months.

Hence, one anomalistic month = S * average synodic month/A =
27;47,11,5,7,21 days

Solar time	synodic month	zodiacal	dracontitic	anomalistic
19 years = 939 3/4 days.	235 months	254		
	29;31,51,3,50 days	27;19,18,39,41 days		
about 18 years 11 days	223		242	239
			27;12,44,19 days	27;47,11,5,7 days

Babylonian Astronomy:

The Uruk Calendar (also called the Metonic Calendar):

The Uruk calendar used throughout the Babylonian world, was the basis of the Jewish Calendar, and was probably exported to Greece at the end of the 4th century BCE. It co-ordinates two phenomena, the length of the year and the length of a lunar synodic month. Now there are more than 12 such months in a year, but less than 13. Let us examine the problem. Suppose that you observe that 235 months corresponds to 19 years, and that there are 365 1/4 days in a year. From here the problem is just mathematical. Hence:

19 years = 235 months = 6939 3/4 days

1 month = 29.5308510638298 days = 29;31,51,3,49,47,14,2,33 days,

The value that appears most commonly is:

1 month = 29;31,51,3,49,47,14,2,33

Obviously, in a civil calendar, a month cannot be 29;31,51 days. Hence, some months must be 29 days (**hollow**) and some 30 days (**full**). In an astronomical calendar, however, this may be a little inconvenient. Instead of measuring a month in days, it may be more convenient to use an absolute measure of time that need not correspond to the civil calendar, but from which one may calculate the civil calendar and which approximates days.

The Uruk calendar divides the month into 30 units. Because the Indian descendants of this calendar called the units 'tithi', it is customary today to refer to these units by their Indian name. Hence, we shall say that the synodic month is divided into 30 **tithis**. This solves the problem of distributing the days in the months.

Now, we have to distribute the months in the year. 12 months per year yields 12*19 months = 228 months. Hence, we need to add 7 months in the course of 19 years. The Uruk scheme interpolates months either after the sixth month of the year or after the twelfth month, according to the following scheme:

Years with "a" intercalenate a second Ulul (last month of the year, before the spring equinox)
Years with "kin-a" intercalenate a month after Adar, the sixth month, before the autumnal equinox)

-368	1			-358	11	
-367	2	a		-357	12	
-366	3			-356	13	a
-365	4			-355	14	
-364	5	a		-354	15	
-363	6			-353	16	a
-362	7			-352	17	
-361	8	a		-351	18	
-360	9			-350	19	kin-a
-359	10	a				

Eclipse prediction (simple version):

A lunar eclipse occurs when the earth is between the moon and the sun when the moon is otherwise visible. The moon does not actually disappear but is a deep red. It is impressive, though not as impressive as a solar eclipse, when the moon is between the earth and the sun, and all that is visible of the sun is a glow around a black disk (called 'the corona'). Solar eclipses are not rarer than lunar eclipses. However, because the shadow of the moon on the earth is much smaller than the shadow of the earth on the moon, lunar eclipses are visible over half of the earth, while solar eclipses are only visible over a swath, not much more than 50 or 60 miles wide. For this reason, it is much more difficult to predict a solar eclipse, since one needs to predict not just when the shadow falls on the earth, but where on earth it will fall. That an eclipse occurs on the southern tip of Africa will never be known to a Babylonian astronomer.

Suppose that the moon is being eclipsed. Then the moon is on the ecliptic and is in opposition with the sun (i.e., the earth blocks the light of the moon from the sun, although the true account of an eclipse is incidental to the theory here presented). If 242 draconitc months equals 223 synodic months, then there will be an eclipse of the same sort 223 months later. However, things are not so simple. For 223 synodic months is about 6585 1/3 days. Hence, if the eclipse occurred in Babylon at 4:00 AM, the next eclipse will be at 12:00 PM and will not be visible in Babylon (late nighters in the Yucatan will see it).

The Greeks called the period 223 synodic months a **SAROS** period. Although the name is not Babylonian, the cycle is. If we multiply the SAROS by 3, then the number of days will be 19756 days 3 hours, which is much more likely to be visible. The Greeks called this the exeligmos cycle.

Let's use the SAROS cycle to calculate and predict eclipses. Now an eclipse can occur whenever the moon is near a node and is either at conjunction (new moon) for a solar eclips or opposition (full moon) for a lunar eclipse. For convenience we will treat of opposition, although it will not matter which.

Now every monoth the moon returns to opposition every 29;31,51,3,50 (synodic month) and the moon returns to the same node every 27;12,44,19,39 days (draconitic month). Hence, each the synodic month full moon occurs later than than the moons crossing a given node than it did in the previous month by 29;31,51,3,50 - 27;12,44,19,39 days = 2;19,6,44,11 days.

However, the moon crosses a node every 27;12,44,19,39 / 2 or 13;36,22,9,50 days. Let us suppose that an eclipse can occur if the moon crosses a node no more than 2;19,6,44,11 / 2 days.or 1;9,33,22,5 days from the full moon.

This is to say that an eclipse can occur if the full moon occurs n * 13;36,22,9,50 days ± 1;9,33,22,5 for some number n from some previous perfect eclipse (where the moon is at the node and full at the exact same time).

In principle, this can occur in the following three circumstances (the times are time of full moon - the time of the crossing of the node, if this is negative, the time of crossing the node occurs first):

	First eclipse	First eclipse	Second eclipse	Second eclipse

	minimum (m_1)	maximum (M_1)	minimum (m_2)	maximum (M_2)
1	-1;9,33,22,5	-1;9,33,22,5,30	1;9,33,22,5,30	1;9,33,22,5,30
5	0;51,15,6,50	1;9,33,22,5	-1;9,33,22,5	-0;51,15,6,50
6	-1;9,33,22,5	0;51,15,6,49	-0;51,15,6,49	1;9,33,22,5

These values are readily calculated as follows: Let n be the number of months, Δ the change in the days nodal crossing less the full moon, and p_n the nodal period = 13;36,22,9,50 days. d_1 is the full moon's date for the first eclipse - the date moon's nearest nodal crossing, and d_2 is the full moon's date for the second eclipse - the date moon's nearest nodal crossing to that eclipse. Then,

$$d_1 + (n * \Delta - p_n) = d_2$$

Hence, we may start with the minimum or maximum for d_1 and calculate d_2. If d_2 thus calculated is out of range, we may take the minimum or maximum for d_2 and calculate d_1

1 month: According to the rule, an eclipse can occur in succeeding months iff the node advances from the minimum to the maximum; otherwise, one will be out of the range.

5 months: If we start with -1;9,33,22,5, then -1;9,33,22,5 + (5 * 2;19,6,44,11 - 13;36,22,9,50) = -3;10,21,51. So we start with m_2.

6 months: If we start with 1;9,33,22,5, then 1;9,33,22,5 + (6 * 2;19,6,44,11 - 13;36,22,9,50) = 1;27,51,37,21. So we start with M_2.

One way we can do this (the Babylonian way) is to note that

$$5 \Delta - p_n = -2;0,48,28,55$$

$$6 \Delta - p_n = 0;18,18,15,16$$

Hence, if d_i = -1;9,33,22,5 then the next eclipse is 1 month later and d_{i+1} = 1;9,33,22,5.
If d_1 - 2;0,48,28,55 is in range, then the next eclipse is 5 months later and $d_{i+1} = d_i$ - 2;0,48,28,55.
Otherwise, d_1 + 0;18,18,15,16 is in range, and the next eclipse is 6 months later and $d_{i+1} = d_i$ + 0;18,18,15,16.

Now every synodic month the moon advances by a certain number of days beyond its node. This increase is 29;31,51,3,50 - 27;12,44,19,39 days (the synodic month - the draconitic month) = 2;19,6,44,11 days. On the other hand, the moon will be at some node every 27;12,44,19,39 / 2 or 13;36,22,9,50 days. Hence, an eclipse is possible every time the difference between the synodic month and the draconitic month is nearly a multiple of 13;36,22,9,50 days. Of course, it will not be visible in Babylon if the full moon is during the day.

Intuitively, we can see that these are the only possibilities, since $5 \Delta < p_n < 6 \Delta$.

The following table gives the month of the eclipse within the SAROS period, the difference in days between the synodic and draconitic cycle, the difference in months, and the difference in days. Observe that the difference in days is a zig-zag linear function between 0 and 1;9,33 days.(half the monthly increment), whose increments are each 0;18,20 or six times the increment less 1/2 a draconitic month. For the an eclipse can occur six months after the beginning of the SAROS. Boldface is where the curve switches direction.

month at full moon	days into the draconitic month	days from the moon's crossing the node	months since last eclipse	difference between values in the third column
1	0	0		
7	0;18,20	0;18,20	6	0;18,20
13	0;36,40	0;36,40	6	0;18,20
19	0;55	0;55	6	0;18,20
24	12;30,35	1;5,47	5	**0;10,47**
30	12;48,55	0;47,27	6	0;18,20
36	13;7,15	0;29,7	6	0;18,20
42	13;25,35	0;10,47	6	0;18,20
48	0;7,33	0;7,33	6	**0;3,14**
54	0;25,53	0;25,53	6	0;18,20
60	0;44,13	0;44,13	6	0;18,20
66	1;2,33	1;2,33	6	0;18,20
71	12;38,8	0;58,14	5	**0;4,19**
77	12;56,28	0;39,54	6	0;18,20
83	13;14,48	0;21,34	6	0;18,20
89	13;33,8	0;3,14	6	0;18,20
95	0;15,6	0;15,6	**6**	**0;11,52**
101	0;33,26	0;33,26	6	0;18,20
107	0;51,46	0;51,46	6	0;18,20
113	1;10,6	1;10,6	6	0;18,20
118	12;45,41	0;50,41	5	0;19,25
124	13;4,1	0;32,21	6	0;18,20
130	13;22,21	0;14,1	6	0;18,20
136	0;4,19	0;4,19	6	**0;9,42**
142	0;22,39	0;22,39	6	0;18,20
148	0;40,59	0;40,59	6	0;18,20
154	0;59,19	0;59,19	6	0;18,20
159	12;34,54	1;1,28	5	**0;2,9**
165	12;53,14	0;43,8	6	0;18,20
171	13;11,34	0;24,48	6	0;18,20
177	13;29,54	0;6,28	6	0;18,20
183	0;11,52	0;11,52	6	**0;5,24**
189	0;30,12	0;30,12	6	0;18,20
195	0;48,32	0;48,32	6	0;18,20
201	1;6,52	1;6,52	6	0;18,20
206	12;42,27	0;53,55	5	**0;12,57**
212	13;0,47	0;35,35	6	0;18,20
218	13;19,7	0;17,15	6	0;18,20
224	0;1,5	0;1,5	6	0;16,10

The value at month 224 should be 0, but is 0;1,5 due to error in rounding.

The value in the last column indicates how full and long the eclipse will be. Hence, the best eclipses will be in months 48, 136, and 224.

The table is inaccurate in that it does not account for the variation in the moon's motion in longitude.

The table may also be used for solar eclipses. However, since a solar eclipse may only be seen in a small area, there is no guarantee that the eclipse will only be visible only in the Falkland Islands. No one could accurately predict solar eclipses before Ptolemy.

Prediction of plantary phases:

A phase is essentially a synodic phenomenon. The irregularities that affect it are:

1. The irregular motion of the planetary star. For slow planets this is almost completely determined by its longitude. Hence, a theory of phases can adequately account for this by attending to longitude. For isotachic planets the issue is more complex since they move with the sun and the irregularity is both a function of the earth's motion and the planets. The same is somewhat true of the moon, as one can see from the fact that the anomalistic month is different from the zodiacal month. Here I shall only discuss the simpler problem of the motion of a slow planet.

2. The angle of the planets motion to the horizon

Given one phase, to find the time and location of the next phase.

Let z be the zodiacal year of a planet (counted in years. Then $s = \frac{z}{z-1}$. Now, this means that the sun will travel in this period for more than one year. The excess over the year is $\frac{z}{z-1} - 1 = \frac{1}{z-1}$. Hence, if $z \geq 2$ (as it is for the three slow planets), the next synodic year will occur one year and a fraction later. Since the sun travels 360° in one year, the sun will in fact move $\frac{360}{z-1}$ degrees relative to its position in the previous year. Meanwhile, since the planet moves 360° in z years, it will move $\frac{360\ s}{z} = \frac{360}{z-1}$ degrees, as well. Hence, if we do not account for the anomaly of the planets motion, all we have to do to calculate the time and the planet's position at the next phase is to add these two values in the following way:

year	month	time difference	position	position difference
n	m	$\frac{1}{z-1}$	p	$\frac{360}{z-1}$
n+1	$m + \frac{1}{z-1}$	$\frac{1}{z-1}$	$p + \frac{360}{z-1}$	$\frac{360}{z-1}$

Obviously, one has to adjust for when $p + \frac{360}{z-1} > 360$ that one subtracts 360, and when $m + \frac{1}{z-1}$ exceeds the length of the year that one take n+2 and subtract the appropriate amount from $m + \frac{1}{z-1}$.

This would be completely adequate but for one crucial problem. The planet and the sun sometimes moves faster and sometimes slower. There are two solutions to this problem.

SYSTEM A

Suppose that we take two or more points on the zodiacal circle. It is most convenient simply to divide it in two segments. Now we assume that the planet moves with one speed in one half of the circle and the other speed in the other half. As for the motion of the sun, we need only be concerned with its speed from positions p to $p + \frac{360}{z-1}$, so that the two speeds may be treated together. The trick is then to add or subtract a constant value from the time difference $\frac{1}{z-1}$ and to add or subtract a constant value from the position difference $\frac{360}{z-1}$. Obviously, one value depends on the other.

Suppose we find that Jupiter completes 22 cycles in 259 years. Then it completes 259-22 = 237 synodic cycles in the same period. 360 (259/237 - 1) = 33˚25 on average. Note that the sun moves 360 (259/237) on average. When the planet is in Gemini 0˚ to Sagittarius 0˚, one might subtract 3˚, and when it is Sagittarius 0˚ to Gemini 0 one might add 3˚. This means adding 36˚25 or 30˚25, awkward numbers. Instead, the trick is to add 30 for 155˚ and 36 for 205˚, whose average is exactly 33˚25.

Hence, when the planet is in one part of the zodiac, one adds 30˚, and when it is in the other part one adds 36˚. What does one do when it crosses the divide. Let the divide be a p, and let the intial position be λ. Let δ be the motion before the divide and d the motion after. Then one notes that up to p, it moves a certain part of the cycle, $c = \frac{p-\lambda}{\delta}$. Hence, the rest of the cycle will be 1 - c, so that it will move $(1 - c) * d = \frac{d}{\delta}(\lambda + \delta - p)$. Verify this.

SYSTEM B

System B treats the motion of the planet as changing according to a linear procedure. The function which it in effect follows is called a zig-zag function. Instead of adding fixed amounts in each cycle, one continuously adds more or less in each cycle. A restriction, therefore, is that the length of the cycles must be equal. Here, there will be a maximum change and a minimum and the amount of change at each occurrance. For example (see ACT 702), one might have the change in the increment be 12' and vary between 11˚14,2,30 and 14˚4,42,30. The result here is that one adds a different amount every time. The mean amount here is 12˚39,22,30 of motion per synodic year. Can you calculate the zodiacal year from this?

When one adds the amount to get a number greater than the maximum (here 14˚4,42,30), one does essentially the same thing as in System A. One calculates the fraction to the maximum amount. Then one subtract the remaining part. Let M be the maximum, δ the amount added to or subtracted from the increment, and Δ the last increment. Then $\Delta + \delta > M$. Hence, the amount of δ that is left to be used is $\Delta + \delta - M$. Since we are now entering the zag part of the zig-zag procedure, we are now subtracting the amount from M, i..e $\Delta_{new} = M - (\Delta + \delta - M) = 2M - \Delta - \delta$. Now we substract δ from the previous value, until $\Delta - \delta < m$, where m is the minimum value. Here, the amount left over now is $\delta - (\Delta - m)$, and we add this amount to m, so that $\Delta_{new} = 2m - \Delta + \delta$.

These two methods existed side by side. One might think that System A is less sophisticated than System B. However, System A is also more flexible in some ways. For example, with System A it is possible to divide the zodiacal circle into several different sections. It is thus possible to have greater variety in the motion of the body. What is clear from these models is:

1. Times and locations can be calculated by the same methods from the same data. The change of time is a trivial conversion from the change of location.
2. If the change of time is fundamental, then one can use very few observations of location. Observations of times can themselves be fairly crude, two or three cycles (where one cycle is a return of the planet to the same general position at the same phase) of synodic years will establish the basic parameters.
3. The mathematics is much more sophisticated than the astronomy.